WEEKLY TEST TYM TEST - 23 Balliwala
SOLUTION Date 06-10-2019

[PHYSICS]

1. $$a = \frac{g \sin\theta}{1 + \frac{K^2}{R^2}}$$

For disc $\frac{K^2}{R^2} = \frac{1}{2} = 0.5$

For sphere $\frac{K^2}{R^2} = \frac{2}{5} = 0.4$

$a(\text{sphere}) > a(\text{disc})$

$\therefore$ sphere reaches first

2. $v = 36$ km/h $= 10$ m/s

By law of conservation of momentum

$2 \times 10 = (2+3)V \quad \Rightarrow V = 4$ m/s

Loss in K.E. $= \frac{1}{2} \times 2 \times (10)^2 - \frac{1}{2} \times 5 \times (4)^2$

$= 60\,\text{J}$

From law of conservation of momentum, we have

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$\Rightarrow \quad v = \frac{m_1u_1 + m_2u_2}{(m_1 + m_2)}$$

Given,

$m = 2$ kg, $u_1 = 36 \times \frac{5}{18} = 10$ m/s,

$m_2 = 3$ kg, $u_2 = 0$

$$\therefore \quad v = \frac{2 \times 10 + 3 \times 0}{2 + 3} = 4 \text{ m/s}$$

Loss in kinetic energy is

$$\Delta K = \Delta K' - \Delta K''$$

$$= \left\{\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right\} - \left\{\frac{1}{2}(m_1 + m_2)v^2\right\}$$

$$= \frac{1}{2} \times 2 \times (10)^2 - \frac{1}{2} \times 5 \times (4)^2$$

$$= 100 - 40 = 60 \text{ J}$$

3. The moment of inertia (I) of a body about an axis is equal to the torque (τ) required to produce unit angular acceleration (α) in the body about that axis is

$$\tau = I\alpha$$

Given, $\alpha = 4\pi$ rad/s^2, $\tau = 31.4$ Nm

$$\Rightarrow \quad I = \frac{\tau}{\alpha} = \frac{31.4}{4 \times 3.14} = 2.5 \text{ kg-m}^2$$

4. $\frac{2}{3}MR_h^2 = \frac{2}{5}MR_S^2$ or $\frac{R_h^2}{R_S^2} = \frac{3}{5}$ or $\frac{R_h}{R_s} = \sqrt{\frac{3}{5}}$

5. Angular velocity is the vector quantity which represents the process of rotation (change of orientation) that occurs at as instant of time. For a rigid body it supplements translational velocity of the centre of mass to describe the full motion. The line of direction of the angular velocity is given by the axis of rotation and the right-hand rule indicates the direction.

6. If no external force acts upon a system of two (or more) bodies then the total momentum of the system remains constant.

Hence, momentum before collision = momentum after collision

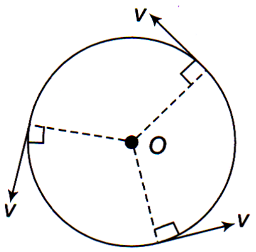
$$m_1 u_1 = m_2 v_2$$

Given, $u_1 = 1$ m/s, $m_2 = 0.05$ kg, $v_2 = 30$ m/s

$$\Rightarrow \quad m_1 \times 1 = 0.05 \times 30$$

$$\Rightarrow \quad m_1 = 1.5 \text{ kg}$$

7. If a body is rotating about an axis, then the sum of the moments of the linear moment of all the particles about the given axis is called the angular momentum of the body about that axis.



$$J = I\omega = mrv$$

Since direction of velocity is perpendicular to orbital plane and $J \propto v$, therefore in an orbital motion the angular momentum vector is perpendicular to the orbital plane.

8. Fractional energy decreases in kinetic energy of neutron

$$=1-\left(\frac{m_1-m_2}{m_1+m_2}\right)^2 \qquad [\text{As } m_1=1 \text{ and } m_2=2]$$

$$=1-\left(\frac{1-2}{1+2}\right)^2=1-\left(\frac{1}{3}\right)^2=1-\frac{1}{9}=\frac{8}{9}$$

9. K.E. of ball in position $B = mg(R-r)$

Here m = mass of ball.

Since it rolls without slipping the ratio of rotational to translational kinetic energy will be 2/5.

$$\frac{K_R}{K_T}=\frac{2}{5}$$

$$K_T=\frac{2}{7}mg(R-r)$$

$$\frac{1}{2}mv^2=\frac{2}{7}mg(R-r)$$

$$v=\frac{2}{\sqrt{7}}\sqrt{g(R-r)}$$

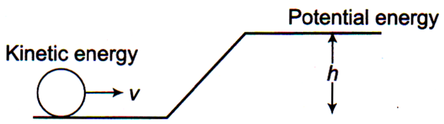
$$\omega=\frac{v}{R-r}=2\sqrt{\frac{g}{7(R-r)}}$$

10. From law of conservation of angular momentum if no external torque is acting upon a body rotating about an axis, then the angular momentum of the body remains constant that is

$$J=I\omega=\text{constant}$$

Hence, if decreases, ω increases and vice–versa. When liquid is dropped, mass increases hence I increases ($I = mr^2$). So, ω decreases, but as soon as the liquid starts falling ω increases again.

11. From law of conservation of energy, energy can neither be created nor be destroyed but it remains conserved. In the given case the sum of kinetic energy of rotation and translation is converted to potential energy.



Also moment of inertia of disc is

$$I = \frac{2}{5}MR^2$$

$$\therefore \quad \underset{\text{(Translational kinetic energy)}}{\frac{1}{2}mv^2} + \underset{\text{(Rotational energy)}}{\frac{1}{2}I\omega^2} = \underset{\text{(Potential energy)}}{mgh}$$

$$\Rightarrow \quad \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\frac{v^2}{R^2} = mgh$$

where $v = R\omega$, ω = angular velocity

$$\Rightarrow \quad \frac{7}{10}mv^2 = mgh$$

$$\Rightarrow \quad v = \sqrt{\frac{10}{7}gh}$$

Hence, to climb the inclined surface the velocity should be greater than $\sqrt{\frac{10}{7}gh}$.

12. Change in momentum = Impulse

= Area under force–time graph

$\therefore$ mv = Area of trapezium

$$\Rightarrow \quad mv = \frac{1}{2}\left(T + \frac{T}{2}\right)F_0$$

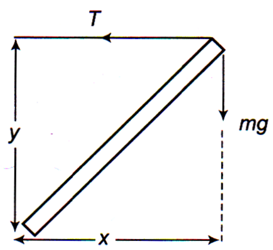
$$\Rightarrow \quad mv = \frac{3T}{4}F_0 \quad \Rightarrow \quad F_0 = \frac{4mu}{3T}$$

13. For equilibrium of street light,

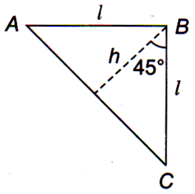
$$mg \times x = T \times y$$

or $$T = \frac{mg.x}{y}$$

For T to be minimum, y should be maximum. Hence, pattern A is more sturdy.



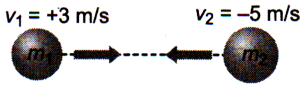
14.



$h = l\cos 45° = \dfrac{l}{\sqrt{2}}$

$I_{AC} = \dfrac{1}{6}Mh^2 = \dfrac{1}{6}M\left(\dfrac{l}{\sqrt{2}}\right)^2 = \dfrac{Ml^2}{12}$

15. As $m_1 = m_2$, therefore after elastic collision velocities of masses get interchanged



i.e., velocity of mass $m_1 = -5$ m/s

and velocity of mass $m_2 = +3$ m/s